"True Commands," and the corresponding jitter metrics are clearly smaller than the nominal values.

Conclusions

The clutter leakage metrics for the measurement of telescope image stability illustrated in this paper are quite valuable, particularly when the telescope undergoes the significant vibrations caused by an adjacent, deformable, articulated body such as a solar array or by control actuators such as reaction wheels or control-moment gyros. In these circumstances, since the Fourier spectrum of the telescope's pointing error is wide, the conventional method of measuring pointing accuracy in terms of stability and stability rates is inadequate. In the numerical example considered, the leakage metric is high for low damping of the solar array, which implies that greater damping yields a better image. Furthermore, the metric depends heavily upon the integration interval; below a certain limit, the smaller the interval, the better the image. The numerical illustration, therefore, also exemplifies control-structure and payload-to-payload interaction.

References

[1]Rapier, J.L., "Clutter Leakage Approximations for Staring Mosaic Senors with Simultaneous LOS Drift and Jitter," Paper 253-32, 24th Technical Symposium, Society of Photo-Optical Instrumentation Engineers, San Diego, CA, July 1980.

[2]Hablani, H. B., "Dynamics of a Deformable Pointing Spacecraft by Kane's Method, and Telescope's Image Stability Evaluation," *Proceedings of the AIAA Guidance, Navigation, and Control Conference,* New York, Aug. 1986, pp. 644–56.

[3]Hablani, H. B., "Design of a Spacecraft Pointing Control System for Tracking Moving Objects," AIAA Paper 87-2597, Aug. 1987.

Principal Coordinate Realization of State Estimation and Its Application to Order Reduction

Yoshikazu Miyazawa* and Earl H. Dowell†
Duke University, Durham, North Carolina

I. Introduction

MODELING of plant dynamics is an important problem, especially when designing a time-invariant optimal estimator. Since the optimal estimator is generally of the same order as the plant dynamics, the estimator becomes of high order if the plant is modeled accurately with a high-order system.[1] In practice, however, such full-state estimation is often useless, and some sort of order reduction is possible and necessary.[2] For the fixed-order optimal estimator problem, optimality conditions and a computational algorithm have been discussed in Refs. 3 and 4. In order to make the iterative algorithm converge to the global minimum, an appropriate initial solution, or a suboptimal reduced-order estimator, is necessary. Furthermore, a quantitative index for measuring the accuracy of the order reduction is desirable.

Received Jan. 14, 1987.

*Visiting Scholar, Department of Mechanical Engineering and Materials Science, on leave from the National Aerospace Laboratory, Japan. Member AIAA.

†Dean, School of Engineering. Fellow AIAA.

This note proposes that a set of singular values be used as an quantitative index that shows estimation accuracy in terms of the state components. Such an approach introduces a unique realization of the plant model and the estimator. Since the singular values show not only estimation quality but also coupling intensities of each state, a principal coordinate realization can be used for the derivation of a reduced-order (or simplified) estimator. The proposed reduced-order estimator does not claim any optimality, nor guarantee better performance than other methods. However, with only a small computational demand, it gives reasonable results, especially when the system has several small singular values.

II. System Description and Principal Coordinate Realization

The plant dynamics is given by the following state equation:

$$\mathrm{d}x/\mathrm{d}t = Ax(t) + Bw(t)$$

$$y(t) = Cx(t) + v(t), \qquad \eta(t) = Dx(t) \tag{1}$$

$x \in R^n$ is the state $y \in R^m$ is the measurement. $\eta \in R^q$ is a variable to be estimated from the measurement y. $w \in R^p$ and $v \in R^m$ are plant disturbance and measurement noise, respectively. It is assumed that they are Gaussian white noise and their stochastic characteristics are given by

$$E\begin{bmatrix} w(t) \\ v(t) \end{bmatrix} = 0, E\left[\begin{pmatrix} w(t) \\ v(t) \end{pmatrix}[w(t+\tau)^T, v(t+\tau)^T]\right] = \begin{bmatrix} W, & W_v \\ W_v^T, & V \end{bmatrix}\delta(\tau) \tag{2}$$

where W and V are positive-definite.

It is assumed that (A,B) is controllable, (C,A) is observable, and the system is asymptoticaly stable. The full-order optimal estimator is given by

$$\mathrm{d}\hat{x}/\mathrm{d}t = A\hat{x}(t) + K[y(t) - C\hat{x}(t)], \qquad \hat{\eta}(t) = D\hat{x}(t)$$

$$K = (XC^T + BW_v)V^{-1} \tag{3}$$

$\hat{\eta}$ is the estimated value for η. The covariance matrix of the error is X, i.e.,

$$E[ee^T] = X, \qquad e(t) = \hat{x}(t) - x(t) \tag{4}$$

It satisfies the following Riccati equation:

$$AX + XA^T + BWB^T - (XC^T + BW_v) \times V^{-1}(XC^T + BW_v)^T = 0 \tag{5}$$

The covariance matrices of the state and its estimate are defined as

$$E[xx^T] = X_1, \qquad E[\hat{x}\hat{x}^T] = X_2 \tag{6}$$

They satisfy the following:

$$AX_1 + X_1A^T + BWB^T = 0, \qquad X_2 = X_1 - X \tag{7}$$

Under the assumptions of controllability and observability, the three covariance matrices are positive-definite, i.e., X_1, X_2, $X > 0$.

When the state transformation $\xi = Tx$ is carried out, the dynamics of the estimator are not changed, and it can be rewritten in terms of the transformed system matrices (TAT^{-1}, TB, CT^{-1}, DT^{-1}). If we consider the transformed covariance matrices of three variables x, $\hat{x}$, and e, there exists a transformation to the coordinate ξ where each covariance matrix is decoupled as follows:

$$E[\xi\xi^T] = TX_1T^T = I_n$$
$$E[\hat{\xi}\hat{\xi}^T] = TX_2T^T = \text{diag}\,(\sigma_1^2, \dots, \sigma_n^2)$$
$$E[\epsilon\epsilon^T] = TXT^T = \text{diag}\,(1-\sigma_1^2, \dots, 1-\sigma_n^2) \tag{8}$$
$$1 > \sigma_1^2 \geq \cdots \geq \sigma_n^2 > 0$$

where $\hat{\xi} = T\hat{x}$, and $\epsilon = Te = \hat{\xi} - \xi$.

σ_i^2 are singular values of the given dynamics and measurement. They show how accurately each state is estimated in terms of its variance ratio. The set of singular values is a quantitative index of system observability, which depends on the disturbance and noise characteristics.

The realization that is obtained by the state transformation could be called a principal coordinate realization of the optimal estimation. The transformation matrix T can be obtained from the simultaneous diagonalization of any two of the three covariance matrices. If the singular values are distinct, the transformation matrix T is uniquely determined. If not, there remains freedom in the choice of coordinate among equal singular values, but the realization that satisfies condition (8) is still possible by arbitrarily defining coordinates.

III. Application to the Reduced-Order Estimator

When the singular values σ_i^2 are nearly equal to zero, the corresponding state is difficult to reconstruct from the measurement. Further, it indicates that truncation of such a state, or simply estimating it to be zero, does not have a significant effect on the overall estimation. Since small values of σ_i^2 also mean that they have small coupling with states with large values of σ_i^2 in the plant dynamics, this realization can be used for the order reduction of the plant and the estimator.

Truncation of less important states introduces a reduced-order estimator, which is given by the following equation:

$$\mathrm{d}\hat{\xi}_r/\mathrm{d}t = A_r\hat{\xi}_r(t) + K_r[y(t) - C_r\hat{\xi}_r(t)]; \qquad \hat{\xi}_r \epsilon R^r\ (r<n)$$
$$\hat{\eta}_r(t) = D_r\hat{\xi}_r(t)$$
$$TAT^{-1} = \begin{bmatrix} A_r, & \times \\ \times, & \times \end{bmatrix}, \qquad TB = \begin{bmatrix} B_r \\ \times \end{bmatrix}, \qquad TK = \begin{bmatrix} K_r \\ \times \end{bmatrix},$$
$$CT^{-1} = [C_r, \times], \qquad DT^{-1} = [D_r, \times]$$

where $\hat{\eta}_r$ is the estimated value for η.

Since the covariance matrices TXT^T and TX_1T^T are diagonalized, the following equations are satisfied:

$$K_r = (X_rC_r^T + B_rW_v)V^{-1}, \quad X_r = \text{diag}\,(1-\sigma_1^2, \dots, 1-\sigma_r^2) \tag{10a}$$
$$A_rX_r + X_rA_r^T + B_rWB_r^T$$
$$-(X_rC_r^T + B_rW_v)V^{-1}(X_rC_r^T + B_rW_v)^T = 0 \tag{10b}$$
$$A_rX_{1r} + X_{1r}A_r^T + B_rWB_r^T = 0, \qquad X_{1r} = I_r \tag{11}$$

The Riccati equation (10b) shows that the reduced-order estimator is the full-order optimal estimator for the following reduced-order plant model:

$$\mathrm{d}\xi_r/\mathrm{d}t = A_r\xi_r(t) + B_rw(t); \qquad \xi_r \epsilon R^r$$
$$y_r(t) = C_r\xi_r(t) + v(t) \tag{12}$$

where the covariance matrix of the state in the reduced-order plant model is given by X_{1r}. The covariance matrices of the reduced-order plant model and the reduced-order estimator are explicitly determined from the diagonalized covariance matrices of the original systems. Since the covariance matrices of the reduced-order systems, X_r and X_{1r} in (10) and (11), are positive-definite matrices, all eigenvalues of the system matrices $A_r - K_rC_r$ and A_r are in the left half-plane, and the stability of the reduced-order estimator and the reduced-order plant model is guaranteed.

When the variable to be estimated is specified as defined by η in (1) and the performance index J is given by the following equation, the importance of each state can be evaluated with weighted singular values by using the concept of component cost.[5]

$$J = E[(\hat{\eta} - \eta)^TQ(\hat{\eta} - \eta)] \tag{13}$$

where Q is an appropriately defined weight. Since the performance index of the optimal estimation satisfies $J = E[\eta^TQ\eta] - E[\hat{\eta}^TQ\hat{\eta}]$, the cost function of the full-order optimal estimator ν can be defined as follows:

$$\nu = E[\hat{\eta}^TQ\hat{\eta}] \tag{14}$$

It can be written as

$$\nu = \sum_{i=1}^{n} \nu_i, \qquad \nu_i = (T^{-T}D^TQDT^{-1})_{ii}\sigma_i^2 \tag{15}$$

where $(\ )_{ii}$ denotes the ith diagonal element of the matrix. By truncating states that correspond to smaller-component cost values ν_i, the reduced-order estimator can be obtained in the same way as given in (9).

IV. Example

As an application of the proposed method, a simple example previously studied in Ref. 2 is considered. The original plant is a fifth-order, single-disturbance, single-measurement system. The plant dynamics is given by the following transfer function:

$$y/w = (s^3 + 12s^2 + 39s + 28)/\Delta(s)$$
$$\Delta(s) = s^5 + 11s^4 + 39s^3 + 61s^2 + 72s + 23 \tag{16}$$

The poles and zeros are $(-5.577, -3.911, -0.5332 \pm 1.443j, -0.4454)$ and $(-7, -4, -1)$, respectively. The noise intensities are given as $W = 10$ and $V = 1$. The singular values obtained are $\sigma_i^2 = 0.9113$, 0.8297, 0.4921, 0.0435, 0.00046. From these figures, it is seen that the last two states are difficult to reconstruct from the measurement. As in the first example of Ref. 2, the performance index is defined as $J = E[(\hat{x}_1 - x_1)^2 + (\hat{x}_2 - x_2)^2 + (\hat{x}_3 - x_3)^2]$, where the transfer functions are $x_1/w = 1/\Delta$, $x_2/w = s/\Delta$, $x_3/w = s^2/\Delta$. The component costs obtained are $\nu_i = 0.4659 \times 10^{-2}$, 0.2835×10^{-2}, 0.1511×10^{-2}, 0.4134×10^{-4}, 0.4394×10^{-8}. In this case, the ranking of each state is the same as that obtained from the singular values. The degradation factor for each reduced-order i, J_{di}, is defined to be the increment of the performance index due to order reduction divided by that of the full-order estimator. The values of J_{di} obtained with the present model are $J_{d4} = 0.1(\%)$, $J_{d3} = 8.0(\%)$, $J_{d2} = 77.0(\%)$, $J_{d1} = 209.0(\%)$. Reference 2 gives J_d for the third-order case as $J_{d3} = 10.5(\%)$, where the method of that reference is based on the optimal reduced-order model of the plant dynamics. The present method gives a little better performance index for this case and is obtained with much lower computational cost.

V. Concluding Remarks

A set of singular values for optimal estimation is proposed. These values provide quantitative indices for system observability and introduce a unique system realization of the plant and the estimator. The realization is utilized for order reduction of the plant and the estimator by simply truncating less important states. The proposed reduced-order estimator gives reasonable performance, especially when the system has some small singular values and the corresponding states are truncated. The proposed method does not require any iterative calculation for optimization, and its computational demand is small.

References

[1]Kwakernaak, H. and Sivan, R., *Linear Optimal Control Systems*, Wiley-Interscience, NY, 1972.

[2]Wilson, D. A. and Mishra, R. N., "Design of Low Order Estimators Using Reduced Models," *International Journal of Control*, Vol. 29, March 1979, pp. 447–456.

[3]Bernstein, D. S. and Hyland, D. C., "The Optimal Projection Equations for Reduced-Order State Estimation," *IEEE Transactions on Automatic Control*, Vol. AC-30, June 1985, pp. 583–585.

[4]Bernstein, D. S., Davis, L. D., and Hyland, D. C., "The Optimal Projection Equations for Reduced-Order, Discrete-Time Modeling, Estimation, and Control," *Journal of Guidance, Control, and Dynamics*, Vol. 9, May–June, 1986, pp. 288–293.

[5]Skelton, R. E. and Yousuff, A., "Component Cost Analysis of Large Scale System," *International Journal of Control*, Vol. 37, Feb. 1983, pp. 285–304.

Book Announcements

STONE, H.W., Carnegie Mellon University, ***Kinematic Modeling, Identification, and Control of Robotic Manipulators,*** Kluwer Academic, Norwell, MA, 1987, 256 pages, $45.00.

Purpose: This book describes the formulation of a new robot kinematic model designed particularly for solving the kinematic parameter identification problem for N degree-of-freedom robotic manipulators with rigid links.

Contents: Review of robot kinematics; identification and control; formulation of the S-model; kinematic identification; inverse kinematics; prototype system and performance evaluation; performance evaluation based upon simulation.

LJUNG, L, Linkoping University, and **SODERSTROM, T.**, Uppsala University, ***Theory and Practice of Recursive Identification,*** The MIT Press, Cambridge, 1987, 552 pages.

Contents: Introduction. Approaches to recursive identification. Models and methods: a general framework. Analysis. Choice of algorithms. Implementation. Applications to recursive identification. Appendices. Index.

BHATTACHARYYA, S.P., Texas A&M University, ***Robust Stabilization Against Structured Perturbations,*** Lecture Notes in Control and Information Sciences, Vol. 99, Springer-Verlag, New York, 1987, 172 pages, $25.10.

Purpose: This book deals with the analysis and design of control systems for plants that contain physical parameters subject to highly structured perturbations.

Contents: The stability hypersphere in parameter space; stability ellipsoids and perturbation polytopes; robust stabilization; structured perturbations in state space models; stabilization with fixed-order controllers; state space design of low-order regulators.

ALONEFTIS, A., ***Stochastic Adaptive Control Results and Simulations,*** Lecture Notes in Control and Information Sciences, Vol. 98, Springer-Verlag, New York, 1987, 120 pages, $20.60.

Purpose: This book presents the direct method of parameter self-tuning control as applied to linear single-input-single-output time invariant systems subject to random disturbances, and its extension to the control of time varying systems.

Contents: Self-tuning control of systems with random disturbances; computer simulations to self-tuning control algorithms; adaptive control of systems with random disturbances; computer simulations of adaptive control algorithms.

PAVELLE, R., Editor, Symbolics, Inc., ***Applications of Computer Algebra,*** Kluwer Academic, Norwell, MA, 1985, 446 pages, $59.95.

Purpose: This volume provides a broad introduction to the capabilities of computer algebra systems that perform numeric and non-numeric computations.

Contents: Macsyma; using Vaxima to write Fortran code; applications of symbolic mathematics to mathematics; stability analysis and optimal control; Fourier transform algorithms for spectral analysis derived with Macsyma; application of Macsyma to kinematics and mechanical systems; derivation of the Hopf bifurcation formula using Linstedt's perturbation method and Macsyma; exact solutions for superlaticces and how to recognize them with computer algebra; computer generation of symbolic generalized inverses and applications to physics and data analysis.

FEATHERSTONE, R., University of Edinburgh, Scotland, UK, ***Robot Dynamics Algorithms,*** Kluwer Academic, Norwell, MA, 1987, 224 pages, $42.50.

Purpose: This book examines methods of calculating the equations of motion for a robot mechanism that can be implemented efficiently on a computer.

Contents: Spatial kinematics; spatial dynamics; inverse dynamics: the recursive Newton-Euler method; forward dynamics: the composite-rigid-body method; forward dynamics: the articulated-body method; extending the dynamics algorithms; coordinate systems and efficiency; contact, impact, and kinematic loops.

La SALLE, J.P., ***The Stability and Control of Discrete Processes,*** Applied Mathematics Sciences, Volume 62, Springer-Verlag, New York, 1986, 150 pages, $22.00.

Purpose: This book deals with the interaction between stability theory and control theory and, connected with that, the stability of dynamical systems.

Contents: Liapunov's direct method; a characterization of stable matrices; computational criteria; Liapunov's characterization of stable matrices; variation of parameters and undermined coefficients; forced oscillations; systems of higher-order equations; the equivalence of polynomial matrices; the control of linear systems; controllability; stabilization by linear feedback; pole assignment; minimum energy control; minimal time-energy feedback control; observability; observers; state estimation; stabilization by dynamic feedback.